218. Proposed by O. W. ANTHONY, DeWitt Clinton High School, New York City.

From a given triangle cnt off an area equivalent to a given square by a line passing through a given point without the triangle.

I. Solution by F. D. POSEY, A. B., Instructor in Mathematics, St. Matthews School, San Mateo, Cal.

Let ABC be the given triangle and P the given point on the opposite side AC say from B. Let a be the side of the given square and suppose the part cut off to be a triangle.

Draw PE perpendicular to AC and PD parallel to AB cutting AC in D. Consider the problem solved and the line PQR cutting AC and AB respectively in Q and R, to be the required line making $AQR=a^2$. Let $AQ\equiv x$, $QD\equiv y$, $PE\equiv m$, $AD\equiv n$. Since AQR and PQD are similar,

$$\frac{x^2}{y^2}=\frac{AQR}{PQD}=\frac{2a^2}{my},\quad \frac{x^2}{y}=\frac{2a^2}{m}.$$

Since $y=n-x$, $\dfrac{x^2}{n-x}=\dfrac{2a^2}{m}$ or $x^2+\dfrac{2a^2}{m}x+\dfrac{2a^2n}{m}$.

Completing the square, $x^2+\dfrac{2a^2}{m}x+\dfrac{a^4}{m^2}=\dfrac{a^2}{m}\left(2n+\dfrac{a^2}{m}\right)$.

Construct a third proportional p to m, a. $\therefore p=a^2/m$.

$\therefore x^2+2px+p^2=p[2n+p]$.

Construct $r=2n+p$ and s a mean proportional between p and r.

$\therefore x^2=2px+p^2=s^2$, $x+p=s$, $x=s-p$.

By constructing $AQ=s-p$, AQR may be shown to have the required area. If the part cut off is not a triangle it may be reduced to the preceding case by cutting off a triangle equivalent to the difference of the given triangle and the given square.

II. Solution by G. W. GREENWOOD, A. M (Oxon), Professor of Mathematics and Astronomy in McKendree College. Lebanon, Ill.

Lines cutting off a triangle of constant area from two tntersecting lines envelope an hyperbola having the given lines as asymptotes.*

Construct hyperbolas for each pair of sides, and draw tangent to them through the given point.

If we are restricted to compass and ruler, we can construct the foci and the auxiliary circle, and by this means construct the tangents through the points.

Other solutions, which arrived too late for insertion, will be printed next month.

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*Take the two lines as the axes of abscissae, the angle between them being w and the given area A. The area of the triangle formed by the axes and the line $x/a+y/b=1$ is $\frac{1}{2}ab\sin w=A$, whence the equation of the line becomes $(ya^2/2A)\sin w-a+x=0$. Differentiating with respect to a and eliminating, the equation of the envelop is found to be the hyperbola $xy=(1/2\sin w)(2A^2-A)$. ED.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ABC be the given triangle, AB base, C the vertex. Let P be the given point without, $PA=b$, $\angle PAC=\theta$, $\angle BAC=A$, area of square$=m^2$, area of triangle$=\Delta$, $\Delta-m^2=n$. Let the line from P cutting off the area m^2 meet AC in D, AB in E; $AD=y$, $AE=x$.

Then $\frac{1}{2}xy\sin A=m^2$ ....(1).

$\frac{1}{2}by\sin\theta=\text{area}PAD$ ....(2).

$\frac{1}{2}bx\sin[\theta+A]=\text{area}PAE$ ....(3).

$\therefore \frac{1}{2}bx\sin[\theta+A]-\frac{1}{2}by\sin\theta=m^2$ ....(4).

The value of y from (1) in (4) gives

$$x^2-\frac{2m^2x}{b\sin[\theta+A]}=\frac{2m^2\sin\theta}{\sin A\sin[\theta+A]}.$$

$$\therefore x=\frac{m^2}{b\sin[\theta+A]}\pm\sqrt{\frac{2m^2\sin\theta}{\sin A\sin[\theta+A]}+\frac{m^4}{b^2\sin^2[\theta+A)}},$$

$$y=\frac{\sin[\theta+A]}{\sin\theta}\left[\pm\sqrt{\frac{2m^2\sin\theta}{\sin A\sin[\theta+A]}+\frac{m^4}{b^2\sin^2[\theta+A]}}-\frac{m^2}{b\sin[\theta+A]}\right].$$

If m^2 is to be the quadrilateral part of the triangle, substitute n for m^2 in the above values for x, y.

Two geometrical constructions which arrived too late for insertion will be printed next month.

219. On account of the pedagogical importance of this problem, the solutions are withheld until next month in order that they may all be printed in the same number.

CALCULUS.

Problem 173 was also solved by L. E. Newcomb, Los Gatos, California.

174. Proposed by B. F. FINKEL, A. M., 204 St. Marks Square, Philadelphia, Pa.

Integrate $\int_0^\infty \frac{\sinh px}{\sinh qx}\cos rx\,dx$, if $p^2<q^2$.

I. Solution by WILLIAM HOOVER, Ph. D., Professor of Mathematics, Ohio State University, Athens, O.

$$I=\int_0^\infty \frac{\sinh px}{\sinh qx}\cos rx\,dx=\int_0^\infty \frac{e^{px}-e^{-px}}{e^{qx}-e^{-qx}}\cos rx\,dx \quad\text{..................(1).}$$

In (1), put $qx=\pi z$, or $x=[\pi/q]z$; then (1) is

$$I=\frac{\pi}{q}\int_0^\infty \frac{e^{az}-e^{-az}}{e^{\pi z}-e^{-\pi z}}\cos mz\,dz \quad\text{..................(2),}$$

in which $a=p\pi/q$, $m=r\pi/q$, and the required integral